It is fairly straightforward to evaluate J_0 and J_1. These are

$$J_0 = 0 \tag{14}$$

$$J_1 = \int_0^\pi \frac{\sin\theta - \sin\phi}{\cos\theta - \cos\phi} d\theta = \int_0^\pi \cot\left(\frac{\theta+\phi}{2}\right) d\theta$$

$$= 2 \log \cot(\phi/2) \tag{15}$$

Hence we conclude from Eq. (13) that

$$A = 2 \log \cot\phi/2 - 4/\sin 2\phi$$

$$B = 0 \tag{16}$$

Thus

$$J_n = 2\frac{\sin n\phi}{\sin\phi} \log \cot(\phi/2) - 4\frac{\sin n\phi}{\sin 2\phi} \quad (n \text{ even}) \tag{17}$$

When n is odd

$$J_{n+1} + J_{n-1} = 2\cos\phi \; J_n - 4/n \tag{18}$$

The left-hand side has even indices, hence using Eq. (17) we have

$$J_n = 2\frac{\sin n\phi}{\sin\phi} \log \cot(\phi/2) - 4\frac{\sin n\phi}{\sin 2\phi} + \frac{2}{n\cos\phi} \quad (n \text{ odd}) \tag{19}$$

Combining Eqs. (17) and (19) we have

$$\int_0^\pi \frac{\sin n\theta}{\cos\theta - \cos\phi} d\theta = 2\frac{\sin n\phi}{\sin\phi} \log \cot\frac{\phi}{2} - 4\frac{\sin n\phi}{\sin 2\phi}$$

$$+ \left[1 - (-1)^n\right] \frac{1}{n\cos\phi} \tag{20}$$

With Eqs. (5) and (20), the solution of Eq. (3) is complete.

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Technical Comments

Comment on "An Inverse Boundary-Layer Method for Compressible Laminar and Turbulent Boundary-Layers"

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THE inverse boundary-layer calculation is one of the classic goals of aerodynamics and the present paper is worthy of the subject. There are only a few minor points requiring clarification. The first point is the matter of turbulent separation prediction which the author feels can be adequately handled by the Cebeci/Smith boundary-layer computation down to zero skin friction; the experimental basis for this statement is said to be presented in Ref. 1. The second point is the author's statement that the capability does not exist today for calculating partially separating flows in two dimensions.

It is a fortuitous coincidence that we have recently presented a methodology[2] for the prediction of the pressure distribution on two-dimensional airfoils with massive turbulent separation; excellent experimental agreement was achieved for the NACA 63-018 at 18° angle of attack, for the NACA 65, 2-421 at 20° and for the NASA GA(W) – 1 at 21°. Our algorithm employed the Cebeci/Smith boundary-layer method, and it was found that it did not yield good results when using the zero skin-friction separation criterion as compared to the test data and to the algorithm with the Goldschmied[3] maximum pressure-recovery criterion.

It is also pointed out in Ref. 2 that meaningful separation prediction requires two parameters, i.e., chordwise location *and* pressure coefficient, while the evidence of Ref. 1 is concerned only with chordwise location. This can be further illustrated in Fig. 8 of the present paper: with the direct method, C_f does not go to zero (no separation) if the correct pressure distribution is employed, while with the inverse method a steeper pressure distribution is required to make C_f reach zero (separation) at the correct location.

The Cebeci method does not allow to predict *both* the correct skin-friction *and* the correct pressure coefficient when separation is present. If the data of Fig. 8 are used to verify directly the separation criterion of Ref. 3, since both skin-friction coefficients and pressure distribution were measured, it is found that good agreement exists for both pressure recovery and axial separation location.

Using the experimental plots of Strickland and Simpson,[4] I locate the minimum pressure point at $X=65$ in. with $U=85$ fps (Fig. 3-3, p. 30) and the corresponding turbulent skin-friction coefficient C_f between 0.0032 and 0.0034 (Fig. 3-30, p. 66), yielding a predicted maximum pressure-recovery coefficient C_{ps} between 0.64 and 0.68. The minimum velocity shown in Fig. 3-3 is $U=51$ fps, yielding the experimental $C_{ps} = 1-(51/85)^2 = 0.64$.

From Fig. 3-30, experimental separation is located between $X=140$ in. and 150 in.; intersecting the inviscid attached pressure distribution at $C_{ps}=0.64$ and 0.68, the axial separation location is obtained between $X=140$ in. and 146 in.

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